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Utilization of Holonomic Distribution Control for Reactionless Path Planning

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***Abstract*— This article introduces a new technique for planning reactionless paths to a point in Cartesian space, for manipulators mounted on a free-floating satellite. It is based on decomposition of the manipulator joint space into sets referred to as *primitives* which have redundancy *one* with respect to the attitude motion of the base body. The time duration of the manipulator motion is divided into sub-intervals. During a given sub-interval only one *primitive* is used. The choice of feasible sequence of *primitives* and times for their actuation, that satisfies given path constraints is made using mixed-variables optimization solver based on a *mesh adaptive direct search* algorithm.**

I. Introduction

Utilization of manipulator systems mounted on free-floating satellites creates a number of technical challenges related to dynamics and control. Some of them are related to the disturbance of the spacecraft's attitude and position as a result of the manipulator's motion. Since such disturbances can lead to problems from the viewpoint of communication and pointing accuracy they are mostly undesirable. Therefore, the dynamic coupling between the arm and spacecraft motion, leads to the necessity for creation of new algorithms for motion planning and control. Even though the spacecraft's motion can be controlled using gas-jet thrusters, this might considerably shorten the systems life. Hence, the utilization of such technique for attitude stabilization has to be limited only to the cases when absolutely necessary.

Trajectory planning for systems under nonholonomic constraints is a well known research field. A robotic manipulator mounted on a free-floating satellite exhibits nonholonomic behavior as a result of the nonintegrability of the angular momentum equation. Up until now, different solutions to the path and trajectory planning problems for space manipulators have been proposed. The concept of the generalized Jacobian matrix was introduced in [1]. It can be used for continuous control of the end-effector without controlling the vehicle's motion. A bidirectional approach for motion planning of free-floating space robots was proposed in [2]. It was shown that the final values of the state variables describing the system, depend not only on the n joint variables but also on the history of their trajectories and do not remain confined on a n-dimensional manifold. Such result clearly implies that, the end-effector can reach a desired position and orientation with different values of the state variables, even if only six joint are available. This indicates the presence of redundancy, in [3] the authors call it nonholonomic redundancy and propose ways for its utilization for facilitating the trajectory planning problem. In [4] Vafa and Dubowsky proposed the novel concept of a *virtual manipulator* which simplifies the kinematics and dynamics of a space robot system. By solving the motion of a virtual manipulator (fixed in the center of mass of the entire system) for a given end-effector trajectory, the motion of the base $\leftrightarrow$ robot arm system can be obtained straightforwardly. Furthermore, using this approach they formulated a tool called "disturbance map" and then extended the notation to an "extended disturbance map" [5], [6] which suggests paths that result in low attitude fuel consumption. Using optimization techniques for performing reactionless trajectory planning, as proposed in [7] does not always converge to satisfactory results, where providing initial guess for the optimization algorithm is of great importance. In [8], the authors propose a manipulator design that provides a larger "reactionless workspace" and address the null space planning problem. In [9], [10], configuration and path planning for nonholonomic systems are discussed. The utilization of optimal control for redundant systems is discussed in [11], [12], [13]. Almost smooth time-invariant control for planar space manipulators is proposed in [14], where the authors discuss a stabilization technique without disregarding the existence of dynamic singularities. In addition, controllability issues related to serial space robot systems are discussed.

This article introduces a new technique (referred to as *holonomic distribution control*) for planning reactionless[1] paths to a desired point in Cartesian space. It has certain similarities with a strategy previously employed for solving the inverse kinematics problem for a redundant manipulator arm, by partitioning the Jacobian matrix into *full rank* minors [15]. The resemblance is in light of the fact that, a decomposition in joint space which leads to certain advantages from the viewpoint of planning and control is made. The main differences are; (i) we consider the system's dynamical characteristics as well; (ii) the joint space is decomposed into sets with redundancy *one* with respect to the base angular motion; (iii) the application considered is reactionless path planning, hence the nature of the problems that need to be dealt with is different. Some of

[1] In this article we focus on the base attitude motion, nevertheless the approach to be introduced can be applied with respect to the base linear motion as well.

them are related to the fact that a free-floating manipulator is a system under nonholonomic constraints; (iv) for the implementation of our approach a *mesh adaptive direct search* (MADS) algorithm is utilized.

The paper is organized as follows. Preliminaries and main notation are presented in section II. The *holonomic distribution control* (HDC) is introduced in section III, and issues related to its application are addressed in section IV. Results from a numerical simulation that demonstrates the usefulness of the proposed planning strategy are presented in section V. Finally the conclusions are summarized in section VI.

II. Preliminaries and Main Notation

A. Basic Equations

We assume that a serial n link manipulator is attached to a free-floating base satellite. The linear and angular velocity of the satellite base $(\boldsymbol{v}_b, \boldsymbol{\omega}_b)$ and the motion rates of the manipulator joints $(\dot{\boldsymbol{\phi}})$ are chosen as generalized coordinates. Assuming that the linear momentum is equal to zero, the angular momentum conservation equation for such a free-floating system can be expressed as follows:

$$\boldsymbol{L} = \tilde{\boldsymbol{H}}_b \boldsymbol{\omega}_b + \tilde{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}} \tag{1}$$

where, $\tilde{\boldsymbol{H}}_b$ and $\tilde{\boldsymbol{H}}_{bm}$ are the base inertia and *coupling inertia* matrices, respectively (for derivation of equation (1) see [16]).

The angular momentum conservation equation (1) is of special interest to us, because it is directly related to the base rotational motion. Attitude destabilization is mostly undesirable, because it can lead to various problems. That is why it will be the primary focus of this article.

In general $\tilde{\boldsymbol{H}}_b$ and $\tilde{\boldsymbol{H}}_{bm}$ are functions of the joint and base variables, however if the external forces and torques are assumed equal to zero, both inertia matrices will depend only on $\boldsymbol{\phi}$. This fact is useful, since with proper joint control, one can ensure minimal base attitude deviation. Such control is referred to as *reactionless manipulation* [17]. Both components on the right side of equation (1) define a partial angular momentum of the system. The first term represents the angular momentum of the base body as a result of its attitude change, the second is related to the manipulator motion and is called the coupling angular momentum between the base and the manipulator [18]. In this context, reactionless manipulation can be defined as such motion that results in zero momentum redistribution between $\tilde{\boldsymbol{H}}_b \boldsymbol{\omega}_b$ and $\tilde{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}}$.

Next, a brief discussion on the types of constraints that need to be dealt with during the planning process, and a short overview of some basic concepts from differential geometry that are needed for the formulation of the *holonomic distribution control*, are made.

B. Pfaffian constraints

In this subsection, a brief treatment of two types of Pfaffian constraints (integrable and nonintegrable) is made. Both appear typically when free-floating systems are studied. It is assumed that the system of interest is *drift free*, in other words the angular momentum $(\boldsymbol{L})$ is equal to zero.

Let us consider a system of holonomic constraints defined by the following set of m algebraic functions:

$$h_i(\boldsymbol{q}) = 0\,, \quad i = 1, ..., m \tag{2}$$

where $\boldsymbol{q} \in R^n$ is a vector that uniquely represents the configuration of the system of interest. After the constraints in (2) are imposed, the motion of the system evolves on a $f = n - m$ dimensional manifold (for a definition of *manifold* see [19] p. 403). In many practical cases, before the application of the constraints in (2), they are reformulated at velocity or acceleration level. Such transition is straightforward and most importantly, reversible ([19] p. 318). Let us write this in the following fashion:

$$h_i(\boldsymbol{q}) = 0 \;\Rightarrow\; \frac{\partial h_i}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} = \nu_i(\boldsymbol{q}) \dot{\boldsymbol{q}} = 0 \;\Rightarrow\; h_i(\boldsymbol{q}) = 0 \tag{3}$$

The set of equations:

$$\nu_i(\boldsymbol{q}) \dot{\boldsymbol{q}} = 0 \tag{4}$$

are called Pfaffian constraints. A set of Pfaffian constraints is said to be integrable, if it is equivalent to a set of algebraic constraints. With *equivalent*, it is implied that the Pfaffian constraints span the same smooth hypersurface in configuration space as the set of algebraic constraints. It is customary to refer to integrable Pfaffian constraints as holonomic, although they are expressed at velocity level, while holonomic constraints are defined by a set of algebraic functions.

In many cases, the formulation of the constraint equations can be done only at velocity level (forming a set of Pfaffian constraint). For example, using equation (1), the condition that should be satisfied in order for a manipulator to perform reactionless motion can be expressed as follows:

$$\tilde{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}} = 0 \tag{5}$$

Note that the above constraint could not be defined explicitly as a function of the manipulator joint positions because the system of interest has a nonholonomic structure, and (in general) for the same manipulator configuration the base position and attitude can be arbitrary.

When a Pfaffian constraint is not defined as the differential of an algebraic function (as in the case of equation (5)), determining weather the resulting constraint is holonomic is not straightforward. A single Pfaffian constraint is said to be *nonholonomic* if it is nonintegrable, hence, it is not equivalent to an algebraic function (defining a holonomic constraint). Discussing integrability in the presence of multiple Pfaffian constraints becomes much more involved. The reason comes from the fact that, even if each of the constraints in (5) is nonintegrable, the combination of two or more of them might lead to an integrable (holonomic) set of constraints. This fact will be fully utilized in the *holonomic distribution control* that will be introduced in the next section.

In many cases, it is convenient to convert a given problem with nonholonomic constraints in another form. By examining the system not from the viewpoint of the *directions of impossible instantaneous motion*, but rather from the viewpoint of the directions in which we are free to move, in other words the *space of allowable motions*. The basis for this space coincides with the right null space (to be denoted by $\aleph$) of the constraints in (5). Hence, the *space of allowable motions* can be expressed as:

$$\dot{\boldsymbol{\phi}} = \aleph_1(\boldsymbol{\phi})u_1 + \aleph_2(\boldsymbol{\phi})u_2 + ... + \aleph_f(\boldsymbol{\phi})u_f \tag{6}$$

where the column vector $\boldsymbol{u} \in R^f$ (containing all the u's) represents the control input of the system in (6), and $\aleph_i$ ($i = 1, 2, ..., f$) are *vector fields* forming the range space of $\aleph$.

At the end of this subsection, it is convenient to adopt some notation from differential geometry, which will be used in Section III for the formulation of the *holonomic distribution control*. The following definitions are adopted from [19].

Definition 1: A *vector field* on R^n is a smooth map which assigns to each point $\boldsymbol{\phi} \in R^n$ a tangent vector $\dot{\boldsymbol{\phi}} \in T_\phi R^n$. Where $T_\phi R^n$ stands for the *tangent space* to point $\boldsymbol{\phi}$.

Definition 2: A *distribution* is a smooth map assigning a linear subspace of $T_\phi R^n$ to each configuration $\boldsymbol{\phi} \in R^n$.

Example for a *distribution* is the linear span of the *vector fields* $\aleph_1(\boldsymbol{\phi}), \aleph_2(\boldsymbol{\phi}), ..., \aleph_f(\boldsymbol{\phi})$ in equation (6). In general we will denote a *distribution* as:

$$\Delta = \mathrm{span}\{\aleph_1(\boldsymbol{\phi}), ..., \aleph_f(\boldsymbol{\phi})\} \tag{7}$$

Evaluated at any point $\boldsymbol{\phi} \in R^n$ the *distribution* defines a linear subspace of the tangent space $T_\phi R^n$:

$$\Delta_\phi = \mathrm{span}\{\aleph_1(\boldsymbol{\phi}), ..., \aleph_f(\boldsymbol{\phi})\} \subset T_\phi R^n \tag{8}$$

Definition 3: A *distribution* Δ is said to be *regular* if the dimension of Δ_ϕ does not vary with $\boldsymbol{\phi}$.

Definition 4: A *distribution* is *involutive* if it is closed under the *Lie bracket*.

III. Holonomic Distribution Control

In this section we propose the *holonomic distribution control* (HDC) in order to simplify the reactionless path planning problem. Its main concept is outlined hereafter.

The HDC is defined to be a control in the form of equation (6), that utilizes a one dimensional *distribution* $\Delta^1 \subset \Delta$. The dimension of the *distribution* Δ coincides with the degree of redundancy of the system, which is $f = n - m^b$, where n is determined by the manipulator joint variables (the system of interest is assumed to be in a free-floating mode), and m^b represents the base task constraints. The reasoning for using only one dimensional *distribution* is based on the fact that, in any configuration $\boldsymbol{\phi}$, the solutions which lead to reactionless manipulation evolve from a one dimensional manifold. By using Lie bracket on the columns of the reaction null space of the coupling inertia matrix $\tilde{\boldsymbol{H}}_{bm}$ (which span the *distribution* Δ), an *involutivity* of Δ^1 can be established [17]. In addition, if the coupling inertia matrix does not loose rank, the *distribution* Δ (and hence $\Delta^1 \subset \Delta$) can be shown to be *regular*. Once *involutivity* of a *regular distribution* Δ^1 is established, its integrability follows directly from the Frobenius' theorem.

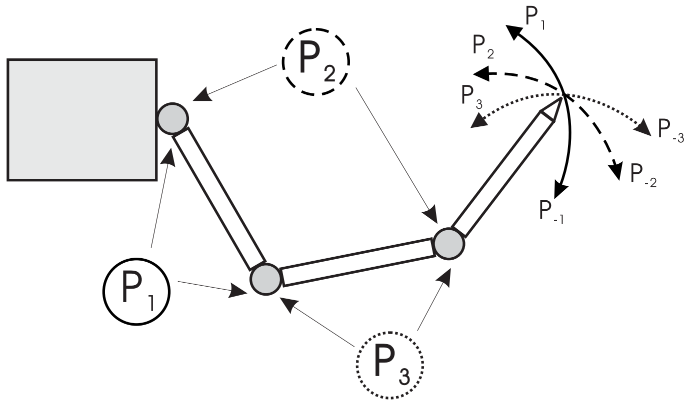

Fig. 1. Cartesian paths when using different *primitives* for a three DOF planar manipulator mounted on a free-floating base.

By choosing different combinations of *vector fields* (members of Δ) in order to form distinct one dimensional *distributions* Δ^1, the motion of the system can be steered in different directions. Furthermore, the constraints corresponding to Δ^1 are holonomic, hence, each of the instantaneously available motion directions, lie on a smooth one dimensional manifold in joint space. Such approach can facilitate the planning problem as will be shown in the sequel.

Remark:

Above, an assumption that the coupling inertia matrix does not loose rank was made. This assumption can be shown to be always valid, if the system of interest has *strong inertial coupling* [20].

One way of defining distinct one dimensional *distributions*, is to partition the manipulator joint variables into a number of sets, referred to as *primitives*. Each *primitive* consists of m^b+1 joint variables (m^b is the number of base rotational motions to be controlled). For example, if a three DOF planar manipulator mounted on a free-floating base is considered, its *primitives* can be defined as depicted in Fig. 1. Since $m^b = 1$, each *primitive* consists of only two joints. For example, primitive 1 is formed by joints 1 and 2. Let us assume that for a time period $t + \Delta t$ just one *primitive* is actuated and the remaining joints are servo locked. Hence, at a given time t each *primitive* defines a direction for the reactionless end-effector motion in Cartesian space[2] (Fig. 1). It is clear that these directions are just a subspace of the possible end-effector reactionless motions from a given manipulator configuration. Nevertheless, the decomposition utilized here facilitates the path planning problem, because manipulator motion derived from one dimensional null space of the coupling inertia matrix $\tilde{\boldsymbol{H}}_{bm}$, results in a curve in Cartesian and joint space (not a surface). Hence, at each manipulator configuration the end-effector motion resulting from a given *primitive* is unique[3].

[2]For the example discussed above there will be six such directions.

[3]In the case when the manipulator motion is derived using a *distribution* with two or higher dimensions, the reactionless paths lie on a two or higher dimensional surface. Choosing a direction on this surface is not a trivial problem, and that is precisely what we want to avoid.

Using HDC regarding systems in three dimensional space is possible. In the case of a 3D five DOF manipulator for example, when $m^b = 3$ five *primitives* exist (each of them consists of four joint variables). In the case of a six DOF manipulator the *primitives* are fifteen.

Once a *primitive* is chosen the joint space is separated into actuated ($\boldsymbol{\phi}^a$) and stationary ($\boldsymbol{\phi}^s$) joints.

$$\begin{bmatrix} \tilde{\boldsymbol{H}}^s_{bm} & \tilde{\boldsymbol{H}}^a_{bm} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\phi}}^s \\ \dot{\boldsymbol{\phi}}^a \end{bmatrix} = 0 \tag{9}$$

The motion rate of $\boldsymbol{\phi}^a$ can be calculated as follows:

$$\dot{\boldsymbol{\phi}}^a = -\tilde{\boldsymbol{H}}^{a+}_{bm} \tilde{\boldsymbol{H}}^s_{bm} \dot{\boldsymbol{\phi}}^s + \aleph^a \dot{\boldsymbol{\xi}}^a \tag{10}$$

where $\aleph^a = (\boldsymbol{E} - \tilde{\boldsymbol{H}}^{a+}_{bm} \tilde{\boldsymbol{H}}^a_{bm})$ represents the null space of the coupling inertia matrix corresponding to the actuated joints ($\tilde{\boldsymbol{H}}^a_{bm}$), $\dot{\boldsymbol{\xi}}^a \in R^{(m^b+1)}$ is an arbitrary column vector, and $\boldsymbol{E}$ is a unit matrix with proper dimensions. If $\dot{\boldsymbol{\phi}}^s = 0$ is assumed, the above equation becomes:

$$\dot{\boldsymbol{\phi}}^a = \aleph^a \dot{\boldsymbol{\xi}}^a \tag{11}$$

It should be noted that, end-effector paths resulting from joint velocities calculated from (11) does not depend on the magnitude of $\dot{\boldsymbol{\xi}}^a$ (assuming that it is not equal to zero). The reasoning for this comes from the fact that the null space of $\tilde{\boldsymbol{H}}^a_{bm}$ is one dimensional. Hence, $\dot{\boldsymbol{\xi}}^a$ can influence only the end-effector velocity on a given path. Since trajectory planning is not the issue here, $\dot{\boldsymbol{\xi}}^a$ will be considered to be with constant magnitude. Nevertheless, its sign can determine the direction of motion and needs to be accounted for.

Using the fact that the Cartesian paths are independent from the magnitude of $\dot{\boldsymbol{\xi}}^a$, the path planning problem reduces to finding a sequence of *primitives*, in combination with durations Δt for the actuation of each *primitive*. A way to determine them will be discussed in the following section.

IV. Holonomic Distribution Control - Application

The problem of finding feasible sequence of *primitives* and times for their actuation that satisfy given path constraints is essential for the successful planning. For simplicity, hereafter only one path constraint will be considered, namely a desired final position for the end-effector. Furthermore, it is assumed that the initial manipulator configuration is known. In this section the reactionless path planning is defined as an optimization problem. The state variables are chosen to be a sequence of h *primitives* $P = [P_i^1, P_i^2, P_i^3, ..., P_i^h]$, and the time for actuation of each of them $T = [\Delta t^1, \Delta t^2, \Delta t^3, ..., \Delta t^h]$, where $i = 0, \pm 1, \pm 2, ..., \pm z$. Note that P_z stands for the last available *primitive* (z), and P_{-z} accounts for the motion in the *opposite* direction[4]. Choosing P_0 will result in a stationary system. It is not necessary to include all *primitives*, in some cases for example, the motion in the *opposite* direction will clearly be unnecessary, hence, it could be disregarded in order to facilitate the optimization solver.

[4]For the case of the three DOF manipulator in Fig. 1 $i = 0, \pm 1, \pm 2, \pm 3$.

Apart from the already mentioned path constraint (final position of the end-effector), the solution to the optimization problem should satisfy the following geometric condition:

$$\boldsymbol{\phi}^{min} \leq \boldsymbol{\phi} \leq \boldsymbol{\phi}^{max} \tag{12}$$

It should be noted that, constraints for the base attitude are not necessary since the manipulator motion is derived from equation (11).

Judging from the state variables (T and P) and constraints defined above, the problem that has to be solved is a typical nonlinear mixed-variables optimization problem. T represent h *continuous* variables, and P represents h *categorical* variables[5].

Solution of a mixed-variables problem can be found using different techniques, here a *mesh adaptive direct search* (MADS) algorithm is utilized. It is very similar to *generalized pattern search* algorithm, however, presents some advantages, since the local exploration of the space of variables is not restricted to a finite number of directions (called *poll* directions). For more details see [21].

Calculation of the manipulator motion and constraints can be performed at kinematical level[6]. For given vectors P and T the precess can be described as follows:

Step ⓪ Initialize counter $j = 1$, and time $t = 0$.

Step ① At time t from the known positions and velocities of the generalized coordinates of the system ($\boldsymbol{r}_b$, $\boldsymbol{v}_b$, $\boldsymbol{\phi}$ and $\dot{\boldsymbol{\phi}}$), compute the coupling inertia matrix $\tilde{\boldsymbol{H}}_{bm}$ (the state variables describing the angular motion of the base are not considered since no attitude change will occur).

Step ② If $t > \Delta t^j$, increment j with one.

Step ③ Use P^j to derive the motion rates for the actuated joints ($\dot{\boldsymbol{\phi}}^a$) from the null space of $\tilde{\boldsymbol{H}}^a_{bm}$.

Step ④ Knowing $\dot{\boldsymbol{\phi}}$ (note that $\dot{\boldsymbol{\phi}}^s = 0$), find the base linear velocity ($\boldsymbol{v}_b$) from the momentum conservation equation.

Step ⑤ Integrate $\boldsymbol{v}_b$ and $\dot{\boldsymbol{\phi}}$ to obtain $\boldsymbol{r}_b$ and $\boldsymbol{\phi}$.

Step ⑥ Increment t with δt (integration step size).

Step ⑦ If $j \leq h$ goto ***Step*** ①.

When the above calculation is over the optimization procedure can evaluate the difference between the real and desired end-effector position as well as the geometric condition (12) and generate new entries for P and T if necessary.

In general, more entries (state variables for the optimization procedure) in P and T result in more precise path planning. The same applies for the number of available *primitives*, since they provide a diversity of the solution at a local level. On the other hand the size of h and z affect the convergence rate of the optimization solver, hence, they should be chosen carefully considering the characteristics of the problem to be solved.

V. Simulation Study

In this section the results from numerical simulation of a 3 DOF planar manipulator mounted on a free-floating base body

[5]Variables whose values must always come from a predefined list. For example, color, shape, or in the case discussed here, *primitive* number.

[6]Since external forces and torques are assumed equal to zero, computation of the system dynamics is not necessary.

TABLE I

MODEL PARAMETERS

	Base	Link 1	Link 2	Link 3
$\boldsymbol{m}$ [kg]	40	2	2	2
$\boldsymbol{l}$ [m]	0.5	1.0	1.0	1.0
$\boldsymbol{I}$ [kgm^2]	25	0.5	0.5	0.5

are presented. The parameters of the system are in Tab. I. The simulation is performed in Matlab 7.0, and the Matlab toolbox Nomadm [22], that implements MADS algorithm is used as an optimization solver.

The results presented hereafter, demonstrate the ability to generate a reactionless Cartesian path for the end-effector from a given initial manipulator configuration to a desired final position. The initial manipulator configuration is taken to be $\dot{\boldsymbol{\phi}} = [15,\ 15,\ -25]$ deg. The desired end-effector final position is assumed to be $[1,\ 1,\ 0]$ m. The available *primitives* are chosen to be (see Fig. 1):

$$i = 0, 1, 2, 3$$

The specified initial guess for the optimization procedure is:

$$P = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]$$
$$T = [8,\ 8,\ 8,\ 8,\ 8,\ 8,\ 8,\ 8,\ 8,\ 8]\ \text{sec.}$$

The vector $\dot{\boldsymbol{\xi}}^a$ used is $[0,\ 0.5]^T$, and the joint limitations are $\boldsymbol{\phi}^{min} = -150$ deg., and $\boldsymbol{\phi}^{max} = 150$ deg. The result from the optimization procedure is:

$$P = [1,\ 2,\ 3,\ 1,\ 2,\ 1,\ 1,\ 2,\ 3,\ 0]$$
$$T = [17.5,\ 12.4,\ 11,\ 6.3,\ 4.4,\ 6.2,\ 14,\ 3.4,\ 2.8,\ 7]\ \text{sec.}$$

If the above sequence of *primitives* in P is used, with time durations the entries of T, the manipulator will reach the desired position in Cartesian space. Fig. 2 depicts three manipulator configurations (initial, intermediate and final one). It can be observed (Fig. 2) that during the motion of the manipulator, the base body undergoes translational motion. This is expected, since only the base attitude was controlled.

The joint angle and joint angular velocity profiles are depicted in Fig. 3. They clearly show the switching from one *primitive* to another. The x axis represents the currently used primitive. The nonsmooth profile of the joint velocities is a result of the constant magnitude of the parameter $\dot{\boldsymbol{\xi}}^a$. Such constant magnitude of $\dot{\boldsymbol{\xi}}^a$ was used in order to facilitate the optimization solver. Once a feasible Cartesian path is obtained however, a smooth joint velocity profile can be generated in a straightforward fashion. Hence, the time profile of the end-effector on the reactionless path can be additionally specified.

In addition, it should be noted that servo locking the joints not included in the currently used *primitive* is just one possible option. Alternatively using predefined profile for their motion can result in a completely different manipulator behavior. This might prove to be useful in cases when the currently available *primitives* cannot provide a desired manipulator motion.

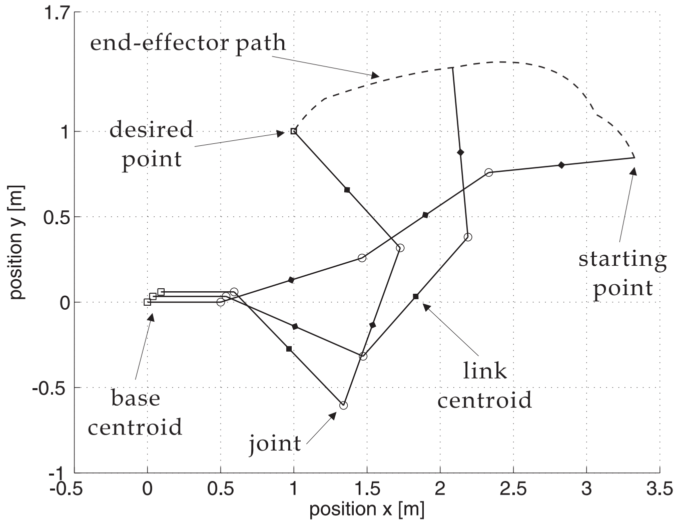

Fig. 2. End-effector path in Cartesian space. The manipulator configuration is depicted at the initial, intermediate and final positions.

After obtaining a solution for P and T, the resultant trajectory might be unsatisfactory. In some cases, discontinuities can be observed during transitions between two *primitives*. This fact though unwelcome is by far not unexpected since only a subspace of the space of possible reactionless motions was utilized. Once a feasible path is obtained however, it can be used as an initial-guess for a new optimization procedure, where criteria regarding the smoothness of a given path segment can be included. This new procedure does not need to use *holonomic distribution control*. Once a good initial-guess is available most of the optimization algorithms can converge to satisfactory results. The merit of the HDC can be found in the fact that it decomposes the entire set of available solutions into small subsets, that can be utilized much easier. If using one subset does not yield satisfactory results it can be changed, and a different one could be utilized. Finding a solution can not be guaranteed since the nature of the problem is highly non-linear, however, the HDC provides a reasonable simplification for the path planning problem. It is worth mentioning that even though the initial guess specified for the example here was trivial, providing a meaningful one is possible. In some cases, experienced user can use HDC and by try and error, reach an adequate solution.

VI. CONCLUSIONS

This paper introduces a new concept (referred to as *holonomic distribution control*) for planning reactionless end-effector paths to a point in Cartesian space. It is based on using a one dimensional *distribution* in joint space by partitioning it into sets called *primitives*. Each primitive consists of $m^b + 1$ manipulator joints, where m^b is the base attitude motions that need to be controlled. Using such approach, the path planning problem reduces to finding a sequence of *primitives*, in combination with times for their actuation, that satisfy a desired criteria. Since each *primitive* results in holonomic

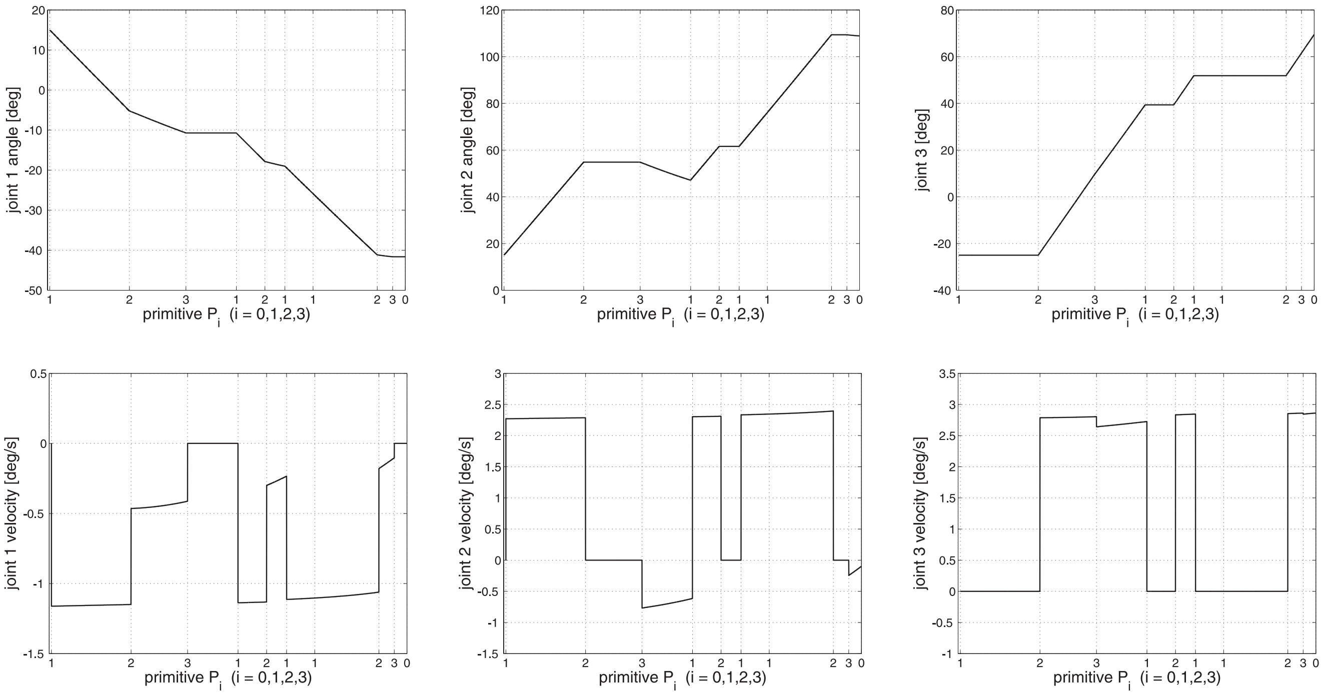

Fig. 3. Profiles of the joint angles and joint angular velocities.

behavior of the system, finding a solution to a nonholonomic planning problem is substituted with finding a feasible sequence holonomic motions.

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